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Uncertainty Quantification Metrics for Whole Product Life Cycle Cost Estimates in Aerospace Innovation

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Abstract

The lack of defensible methods for quantifying cost estimate uncertainty over the whole product life cycle of aerospace innovations such as propulsion systems or airframes poses a significant challenge to the creation of accurate and defensible cost estimates. Based on the axiomatic definition of uncertainty as the actual prediction error of the cost estimate, this paper provides a comprehensive overview of metrics used for the uncertainty quantification of cost estimates based on a literature review, an evaluation of publicly funded projects such as part of the CORDIS or Horizon 2020 programs, and an analysis of established approaches used by organizations such NASA, the U.S. Department of Defence, the ESA, and various commercial companies. The metrics are categorized based on their foundational character (foundations), their use in practice (state-of-practice), their availability for practice (state-of-art) and those suggested for future exploration (state-of-future). Insights gained were that a variety of uncertainty quantification metrics exist whose suitability depends on the volatility of available relevant information, as defined by technical and cost readiness level, and the number of whole product life cycle phases the estimate is intended to be valid for. Information volatility and number of whole product life cycle phases can hereby be considered as defining multi-dimensional probability fields admitting various uncertainty quantification metric families with identifiable thresholds for transitioning between them. The key research gaps identified were the lacking guidance grounded in theory for the selection of uncertainty quantification metrics and lacking practical alternatives to metrics based on the Central Limit Theorem. An innovative uncertainty quantification framework consisting of; a set-theory based typology, a data library, a classification system, and a corresponding input-output model are put forward to address this research gap as the basis for future work in this field.

Keywords: cost estimation; cost readiness; innovation; uncertainty quantification; whole product life cycle

Table of contents

List of tables

List of figures

Definitions and abbreviations

The appendix contains a summary of key terms, abbreviations and definition as raised in the paper in order to ease review by the reader.

Executive summary

This paper aims to provide the cost estimator of novel aerospace products with guidance grounded in theory for selecting the most suitable metrics to quantify the propagation of uncertainty of their estimate. A suitable metric is defined as being one which avoids the need for data normalization in order to achieve statistically significant accuracy. For example when no historical data is available for propagation the probability density function based on the Law of Large Numbers is less suitable than measures of entropy or fuzziness. An intensive review of literature and case studies demonstrates that depending on the length of the forecast window a variety of metrics are available. The figure illustrates the concept explored by highlighting the challenge presented as a progression of questions that need to be addressed.

Figure 1: Uncertainty metric selection

The theoretical foundations for metric selection are based on a definition of uncertainty as a space termed "probability field" within which the cost estimate and the verified cost exist. The difference between the two is declared to represent manifested uncertainty. The set of available metrics is determined through a review of relevant metrics discussed since the mid-16th century and the suitability of such investigated from the degree of reliance on the Law of Large Numbers which is also known as the Central

Limit Theorem. Key insights gained are that metrics used in the cost estimation of novel aerospace products remain based on statistical approaches finding their origin in the late 19th century and that diffusion of metrics suited to latter day contexts can be measured in life-spans versus years. The proposed impact of the presented research effort is an accelerated adoption of uncertainty quantification metrics suited for novel complex low volume and long lifetime products as found in the aerospace industry. The benefit of this accelerated adoption is suggested to be more accurate forecasting of whole product life cycle costs and through this increased financial planning accuracy leading to more sustainable organisational and industry performance. The practitioner may wish to focus on the introduction, context, and framework and metrics as presented in Section 6. Theoretical foundation for the framework and metrics can be examined in the discussion of the review methodology and states of UQ. The dynamics of metric adoption over time can be explored in the critical analysis. The appendix on definitions and abbreviations should be referenced especially in respect to the sections on the states of UQ and their critical analysis.

1 Introduction

Summary of this section:

1. *Uncertainty quantification is about predicting the actual prediction error of a technical baseline estimate.*
2. *Uncertainty quantification is influenced by static and dynamic variables.*
3. *Uncertainty quantification should occur as part of a risk assessment process after determination of the technical baseline estimate.*

4. *Cost estimators have little guidance grounded in theory for choosing Uncertainty quantification metrics.*
5. *Cost estimators focus on pattern matching versus pattern recognition*
6. *The aim of the cost estimator is to minimize the actual prediction error.*

The motivation for this paper lies in concerns voiced by aerospace industry executives regarding the inability to accurately calculate actual unit cost of mature aerospace propulsion systems, the inherent inability to predict the costs of future aerospace innovations and the willingness to accept financial losses on initial production series due to this situation. These concerns were aptly summarized by a finance executive in the statement "The greater the innovation involved the less we know which data is relevant for cost estimation, how to gather it and how to measure it consistently across the whole product life cycle in respect to cost. Cost estimation is an appendix and not an integral component of our life cycle management". In answer to the follow-up question regarding what was actually being measured the executives responded that the focus was on detailed engineering break-downs with corresponding single point estimates by the responsible supply chain units with addition of a small contingency "just in case". Further investigation revealed that multiple methods, techniques, tools and data sources were being used to create (in-) visibility of cost (uncertainty) across the whole product life cycle based on a fragmented information landscape populated primarily by inaccurate data with little effort to support the coherent transition of information across life cycle phases and organizational functions. In latter-day global aerospace corporations with heavily outsourced supply chains and organically grown infrastructures (often through mergers and acquisitions) the challenges to bring cost estimation “under control” are significant to say the least. Conversations with the cost estimation community then also highlighted that:

- little difference was being made between cost and cost “risk” estimation since the cost estimates were primarily gathered as cost commitments from supply chain units, i.e. cost “risk” was delegated deeply into the supply chain without closer consideration of the growing system level risks involved as component aggregation increases,
- cost estimation functions were tasked to meet target / affordability costing limits without being directly involved in the relevant engineering trade-off analyses,
- efforts to integrate cost estimation into the engineering solution landscape using estimation tools often failed due to the high complexity and volatility of the engineering features and manufacturing methods involved.

This literature review is intended to help the estimator and decision maker of today understand what actually constitutes the “uncertainty” of an estimate. The literature review attempts to identify the metrics that are available for describing and quantifying uncertainty under different conditions to provide a basis for more valuable discussion and sense-making from a variety of perspectives. Giving the estimator and the decision maker a wider spectrum of defensible techniques along with the requisite understanding of which problems they might be suitable for solving, gives all involved stakeholders the opportunity to go beyond “tick-box exercises”, raise the estimation (and management) of cost uncertainty to the importance it has and help achieve the significant benefits embodied therein. Figure 2 illustrates the fundamental challenge faced by the estimator as they engage in creating such an estimate in that the requirements for the range of the forecast and the confidence levels needed for decision making, the most suitable metric needs to be chosen to quantify volatile uncertainty.

Figure 2: The uncertainty quantification puzzle

The axiom upon which the investigation is based is that the main objective of uncertainty quantification is to forecast the actual prediction error of the cost estimate as accurately as possible. While the actual prediction error can only be verified once the effort in question has been completed, the deviance-forecast itself occurs in a dynamic and evolving context during the whole product life cycle. . This results in the deviance-forecast requiring continuous adjustment. This adjustment is difficult to predict due to the wide range of potential influencers and the resulting rise in computational complexity [1]. The (un-) changing attributes of uncertainty also exhibit attributes which may be at tension with each other as suggested by Zeno and over time by many other uncertainty principles in the sciences, i.e.

“If you look at an arrow in flight, at a single instance in time the arrow is at some location, and it appears at that instant the same as a

motionless arrow. Then how do we see motion?" Zeno´s paradox [2]

Uncertainty is the arrow in this paradox and uncertainty quantification refers to the determination of the dynamic element of uncertainty. While uncertainty quantification in practice is typically at best considered the result of a risk assessment process based around a technical baseline estimate, this uncertainty typically represents a static snap-shot of current conditions without defensible explanation of future development. It is however the future development of this uncertainty that is critical to understand when it comes to aerospace innovations where the time between estimation and the point of verification can occur may be measured in years and the financial investments involved require short-term decisions which are significant enough to threaten the future of the relevant company if judged wrongly. When determining the uncertainty of a cost estimate the estimator, similar to when creating the cost estimate itself, has several fundamental decisions to make, whereby in general it is perceived that "no one solution is theoretically better than the other ones" [3]. The fundamental questions and the extent the review results might impact them, are highlighted in Table 1. The symbol "✓" indicates that the question is influenced by the research efforts, while the symbol "x" indicates that the question is not explicitly considered.

Table 1. Estimation questions addressed by the states

Question	State-of-practice	State-of-art	State-of-future
Which data should be used?	x	✓	✓
Which metrics should be applied?	x	✓	✓
How should the center of the data / problem of the middle be determined / addressed?	x	x	✓
Which metric should be used describe the center?	x	x	✓
Should pattern-matching or pattern-recognition approaches be used?	x	x	✓

Once these questions have been answered the estimation of uncertainty is the outcome of a computational process which is influenced by further choices related to propagation methods, prediction methods, random sampling methods, and experimental design. The computational complexity class is also relevant from objective and subjective perspectives, whereby the latter not only refers to capabilities, but also to cognitive biases of stakeholders. These decisions must be made by the estimator without defensible theoretical guidance which choices are the most suited for the context of the estimate (i.e. technical or cost readiness level of the product). As discussed by Golkarl and Crawley [4] in respect to distributions for pattern matching "The assumption of a distribution is arbitrary in this context, as there is no firm rationale on how to choose a distribution over another." Indeed established practices of estimators for uncertainty quantification are hence perhaps no more than "conjective inductions that are open to experimentation and testing in search of solutions that describe reality more effectively" as suggested by Miller in Popper´s discussion of "The Problem of Induction" in 1985 [5], addressed by Nolan and Pickard [6] from an industrial perspective, Smithson from a disaster perspective [7, 8] or Priemus et. al. from a decision making perspective [9].

When exploring different approaches for uncertainty quantification it is important to first arrive at an understanding of various perspectives taken in the references identified as illustrated by Table 2. The dominance of the engineering perspective led to the definition of uncertainty as an actual prediction error since such a cost estimate used as the basis for (no-) bid decisions in the whole product life cycle of engineering products. The other perspectives typically focus more on the use of ranges in uncertainty quantification whereby confusion may often arise since the perspectives will mingle in practice.

Table 2. References by uncertainty perspective

Perspective of a reference	**Number of references**
Engineering	85
Mathematics	47
Risk	29
Policy	23
Finance	1

It is important to remember hereby that uncertainty always increases the magnitude of a technical baseline estimate because the technical baseline estimate is the outcome of a dedicated technically focused estimating process which is then used as the input for a cost risk, or cost "threat" assessment process [10]. The treatment of opportunities which reduce uncertainty is considered to require separate assessment, i.e. a cost opportunity process which finds no explicit consideration in literature. The next question is how we might best describe this error [11]. Error description is hereby dependent on the metric being applied and while a range of potentially suitable metrics exist the literature review insights suggests that uncertainty at different technical and cost readiness levels is best described by different metrics. The less data is available for regression analysis suited for forward propagation, the lower the technical readiness level is by default. The less data is available the less sure we can be that the elicited data will admit a probability density function that is based on the Central Limit Theorem and hence the more we must tend to metrics not dependent on these. The suitability of metrics therefore depends on the amount of data required for defensible pattern recognition, whereby we must remember that most statistical pattern recognition software algorithms in fact use Central Limit Theorem based regression techniques in their algorithms in the first place. On the other hand we might actually argue that the more data we have, the more difficult it becomes to find the right or most relevant pattern in the first place [12, 13].

Section 2 describes the context giving rise to the review and Section 3 provides an overview of the research methodology used. Section 4 provides a detailed overview of the field of uncertainty quantification and Section 5 shares a critical analysis of results. Section 6 discusses the research gap while Section 7 shares the conclusions and recommendations for future work. The practitioner may wish to focus on the introduction, context, and framework and metrics. Theoretical foundation for the framework and metrics can be examined in the discussion of the review methodology and states of uncertainty quantification. The dynamics of metric adoption over time can be explored in the critical analysis. The appendix on definitions and abbreviations should be referenced especially in respect to the sections on the states of uncertainty quantification and their critical analysis.

2 Context

Summary of this section:

1. *Uncertainty quantification prediction errors in practice are significant.*
2. *Cost estimates are usually single point plus contingency.*
3. *The system of relevance is complex.*
4. *Uncertainty quantification is seldom performed as part of a risk assessment.*
5. *Uncertainty quantification approaches are seldom synchronized across the whole product life cycle.*
6. *Uncertainty management is often delegated to suppliers.*

Aerospace innovations can be understood as "systems-of-systems" (SoS) which are defined as an "interoperating collection of component systems that produce results unachievable by the individual systems alone" [15]. The management of SoS is typically challenged by the interdependent operation of system elements and the different whole product life cycles of these, whereby requirements also mature significantly during the phases leading up to in-service. Management of SoS is also typically a highly distributed complex collaboration task with unclear boundaries and lacking halting rules especially in respect to requirements engineering [16]. Such a SoS view is helpful to understand that all requirements are essentially interdependent and the more advanced a product the more the development of new requirements over the product life cycle dominates the uncertainty calculation. This is especially true in an age where the growing interdependence of airframes with their (sub-) assemblies / components, such as the propulsion systems, and relevant industry infrastructure continuously raises new challenges [17, 18, 19, 20, 21, 22]. Add to this the observation that, as in many other industries, the time between concept, prototype and operations is shrinking rapidly and we begin to sense why the accuracy of very early stage cost estimates play an increasingly significant role not only in the decision about whether to pursue opportunities, but also in the management of cost as a whole during the (shrinking) life cycles. In parallel the maturing concept of the engineering product service system provides insights into how these relational complexities might best be dealt with [23, 24].

Reflecting on this context, while the expectation is that an engineering break-down cost approach should achieve the most accurate cost estimation it most often does not. Over the past decades alternatives, such as parametric estimation techniques [25], have thus arisen in order to compensate for this situation although the accuracy there remains, as in the engineering breakdown approach, heavily dependent on the existence of sufficient historical information for regression analysis and normalization. Gathering sufficient data is an expensive effort that takes significant time investment by experts while in many cases even then being thwarted by the lack of data in the first place. The question raised hence becomes all the more valid: what can be measured and what metrics should be applied? State-of-practice cost estimation approaches by default assume that cost estimation data is available, will follow the law of large numbers and present standard probability density functions to which normalization can fit the data available so that it represents future reality in a defensible manner. Practice seems to indicate that this assumption only holds for aerospace innovations of high technical and cost readiness however, i.e. the more units are produced and brought in-service the more data is available for cost evaluation of incremental changes. Considering that the fly-away costs of the first units are significant (especially since these will include non-recurrent engineering costs), finding a different approach to quantifying the potential uncertainty of early cost estimates is growing in importance.

The reason why cost estimation for these types of products differs considerably from other products can also be seen by exploring Table 3 based on Haskins [15] where the recommended activities for cost estimation are compared between traditional products and aerospace innovation. The column indicating aerospace innovation attributes is based on the literature review and added by the researchers.

Table 3. Traditional product versus aerospace innovation cost estimation activities

Activity	Traditional Product	Innovative Product
1. Obtain a complete definition of the system, elements, and their subsystems (SoS).	Requirements largely defined and understood based on market maturity of earlier products.	Requirements only partially known and high volume of changes expected far into the life cycle due to lacking experience / historical data.
2. Determine the total number of production units of each element to develop parametric cost data for operations.	Product order magnitude large since application, reliability etc. are clear with low uncertainty in respect to performance.	Product order magnitude low since actual performance is unclear.
3. Obtain the life cycle program schedule.	Schedule is the "standard" schedule with experience in managing it.	Schedule is "standard" however there is significant uncertainty in respect to how long the various phases will last.
4. Obtain manpower estimates for each phase of the entire program and, if possible, for each element and subsystem.	Estimates based on operational experience with very similar products.	Estimates difficult to provide due to novel requirements.
5. Obtain approximate/actual overhead, general and administrative burden rates and fees that should be applied to hardware and manpower estimates.	Overheads generally known.	Overheads not necessarily impacted – no difference to traditional product.

6. Develop cost estimates for each subsystem of each system element for each phase of the program.	Relatively reliable historical data with low uncertainty ranges is available.	Little or no historical data is available in the first place.

This systems engineering perspective is mirrored in various approaches taken from the perspectives of whole life product cycle costing [26, 27, 28, 29, 30] whereby special consideration is often given to concurrent engineering approaches [31], very large projects [32] or mathematical theory [33].

3 Review methodology

Summary of this section:

1. *The discussion of uncertainty quantification is highly fragmented in research and practice.*
2. *Little differentiation is made between uncertainty quantification typologies, frameworks, methods, techniques and metrics.*
3. *Best-learning is provided by Joint Agencies, NASA, USAF, RAND and INCOSE.*
4. *The spectrum of available potential uncertainty quantification metrics is typically not seen / understood.*
5. *Uncertainty quantification metric families are point, range, shape, homogeneity, compression and complexity.*
6. *Each metric family consists of specific and generic metrics suited for uncertainty quantification.*

The literature review builds on the review of a series of fundamental references in the field of aerospace innovation related cost estimation in order to determine the uncertainty quantification approach taken. From a state-of-art perspective the foundational literature is the “Joint Agency Cost

Schedule Risk and Uncertainty Hand Book" [186], the "NASA Cost Estimating Handbook" [10], the INCOSE "Systems Engineering Handbook" [15], the "Handbook of Cost Risk Analysis Methods" [34], the RAND Report "Evaluating Uncertainty in Cost Estimates" [35], and the "Cost Risk and Uncertainty Analysis Handbook" [36]. Based upon this review of the state-of-art references, key words were identified and used as the basis for literature research in online databases such as Scopus. The Web of Knowledge was used to explore citation maps in more details as a way of understanding where important aggregations of thinking appear to occur. Each identified source was manually text-mined to identify metrics of relevance.

Nine journals were identified with more than one contribution relevant to the research study: Progress in Aerospace Science (10 contributions), Risk Analysis (5 contributions), International Journal of Production Research (3 contributions), Journal of Engineering Design (3 contributions), Technological Forecasting & Social Change (3 contributions), International Journal of Advanced Manufacturing Technology (2 contributions), International Journal of Computer Integrated Manufacturing (2 contributions), International Journal of Project Management (2 contributions), Systems Engineering (2 contributions).

Forty-seven other journals that contained individually relevant articles were Acta Astronautica, Chance, Climatic Change, Communications in Statistics - Theory and Methods, Cost Engineering, Geophysics, Global Environmental Change, Harvard Business Review, IEEE Transactions on Engineering Management, Industrial and Commercial Training, Information and Control, International Journal of Production Economics, International Journal of Services Operations and Informatics, JDMS, Journal for Multivariate Analysis, Journal of Aerospace Engineering, Journal of Cost Analysis and Parametrics, Journal of Cost Analysis, Journal of Cost Management, Journal of Intelligent Manufacturing, Journal of Management Science, Journal of Manufacturing Science and Engineering, Journal of Mechanical Design, Journal of Product Innovation Management, Journal of the Royal Statistical Society, Journal of Uncertainty Quantification, Judgment and Decision Making, Management Science, Management Sciences and Global Strategies in the 21st Century, Mathematische Annalen, Neurocomputing, Physical Review, Policy Sciences, Psychological Methods, RAND Technical Reports, Reviews of Moderrn Physics, Science of the Total Environment, Scientific American, Statistical Science, Structural and Multidisciplinary Optimization, Systems Research, Technovation, The Bell System Technical Journal, The Journal of Computational Statistics and Data Analysis, The Philosophical Review, Transport Economics, and Tunnelling and Underground Space Technology.

Multiple conferences were identified as sources for papers although here no specific emphasis on a single event could be identified. In addition to the 83 journal articles, 20 conference contributions, 46 industrial handbooks / reports / standards and guides, 8 PhD theses, 43 case studies and 28 books were reviewed. Table 4 provides an overview of literature review source type frequencies.

Table 4. Literature source type frequencies

Source Type	Frequency
Journals	83
Guides	46
Case Studies	43
Books	28
Conferences	20
Theses	8

Table 5 provides an overview of literature review sources with frequencies of greater than 1.

Table 5. Overview of literature review source frequency

Source	Frequency
Progress in Aerospace Science	10
Risk Analysis	5
Technological Forecasting and Social Change	3
International Journal of Production Research	3
Journal of Engineering Design	3
International Journal of Advanced Manufacturing Technology	2
International Journal of Computer Integrated Manufacturing	2
International Journal of Project Management	2
Systems Engineering	2

Relevant literature was determined through a focus on whether uncertainty metrics were the subject of discussion versus uncertainty determination methods or meta-data (such as the concept of the probability density function). Of note is that no literature sources were found which explicitly discuss the merit of different metrics for uncertainty quantification. Since very little literature was identified that directly discussed the availability and (comparative) suitability of various metrics, literature that at least touched upon specific metrics was considered. Further filter criteria were the ability to retrieve the information from a stable peer-reviewed source (i.e. journals), and emphasis on primary research sources (although some secondary sources were used due to inability to review primary source for language, access or complexity reasons).

4 States of Uncertainty Quantification

Summary of this section:

1. *Uncertainty quantification metrics in literature can be identified and categorized.*
2. *Mathematics and the natural sciences are the birthplaces of uncertainty quantification metrics.*
3. *Uncertainty quantification metrics live in probability fields.*
4. *The history of uncertainty quantification can be understood as separated into foundations, state-of-past and state-of-art.*
5. *The state-of-practice and state-of-art are best described by case studies and industry guidelines.*
6. *The state-of-future is best described by journal contributions.*

The review of uncertainty quantification metrics is based on an initial definition of probability fields as the context of relevance (Section 4.1), the Central Limit Theorem as foundational concept (Section 4.2), a review of historical developments in the field (Section 4.3), a detailed presentation of uncertainty quantification metrics identified by literature source (Section 4.4) followed by a review of state-of-practice (Section 4.5), state-of-art (Section 4.6), a comparison of state-of-practice to state-of-art (Section 4.7) and state-of-future (Section 4.8).

4.1 Probability fields

The range of values a single point technical baseline estimate may have and the probability of the magnitude of these values is described by uncertainty quantification metrics and considered a multi-dimensional probability field. This probability field and its associated values can be interpreted as uncertainty spaces [37], Hilbert spaces [38, 39] probability fields [40] or hyper-spheres [41] and may change over time as the variables influencing it change. The probability field is defined by lower and upper boundaries which are set by subjective threshold parameters, i.e. desired confidence levels. The desired probability field is the smallest range containing both estimate and verified value. From this perspective boundaries are not necessarily linear and may be defined by polynomial and scenario sensitive functions.

The probability field of the single point technical baseline estimate generated by a cost estimation process represents a zero dimensional point consisting of the expected cost at 100% probability for the point in time being estimated for. The cost risk process uses the single point technical baseline estimate as the lower bound (assuming only threats which increase cost are evaluated) and identifies an upper cost bound at a 100% confidence level. The progression to the 100% confidence level is described by the cumulative density function. The cost risk process adds a cost range to generate a one dimensional line on the probability / cost plane. The previous evaluation of cost and cost risk is then expanded to include a spectrum of probability based on the minimum confidence level demanded for decision making and generates a two dimensional space. Since the probability field changes over time this dimension needs to be added. Probability fields typically do not have straight line boundaries and the information distributed within it is not uniform. In this respect Figure 3 illustrates the aggregated sliding windows of an exemplary three dimensional probability fields of anonymized 122 months of enterprise cost risk data using a two dimensional response surface representing most fitting probability density functions which is a Weibull consistently for each time slice) [42, 43].

Figure 3: Adjusted form of three dimensional technical baseline estimate based on most fitting probability density function

Figure 4 then illustrates the aggregated sliding windows of three dimensional uncertainty quantification of this data using a two dimensional response surface based on applying the concept of Shannon density for each time slice.

Figure 4: Sample uncertainty propagation based on Shannon entropy

Uncertainty quantification metrics hence not only need to be able to describe such dynamic response surfaces as they propagate over time, but also be suitable for predicting their development. To some degree we might be reminded of Michelangelo´s perspective that the shape of sculpture already exists in a block of marble and that it is the artist´s task to uncover this shape as truly as possible. Based on this analogy we might then suggest that the shape and the unfolding propagation behaviour of cost uncertainty is deterministic within the fidelity of the effort itself, yet due to the complexity of the probability field a bottom-up predication represents a computational complexity class that is not solvable in polynomial time and parametric efforts also fail due to the lack of knowledge of the needed cost estimating relationships (which may not be discoverable in polynomial time in any case). We might also raise the question whether the pattern which appears “hidden” in the probability field is less related to the information distribution itself, and more to the manner in which this evolves / emerges or the rules which apply to this.

What remains is the critical question whether the techniques and metrics commonly used in this context are sufficient, or whether perhaps other techniques and metrics exist which are more suitable for uncovering and forecasting the propagating cost uncertainty patterns over time. It is particularly this dynamic propagation which may admit principles developed from the perspectives of cellular automatons [44, 45] or similar self-organising living systems.

4.2 The role of the Central Limit Theorem

A fundamental question addressed by the researchers is that of when the Central Limit Theorem can be used defensibly to determine the probability of an event occurring. The Central Limit Theorem essentially states that given a sufficiently large number of observations, i.e. 10 000 as used in a Monte Carlo simulation, the probability distribution of events will follow a single modal bell-curve pattern. Each observation must hereby be randomly generated to ensure that there is no dependency between the values of observations gathered.

The Central Limit Theorem primarily describes the behavior of the single center of the data and is a special case of the law of large numbers which proposes that if an experiment is conducted a sufficient number of times the average result of the experiment will normalize to a single value. The key reason for this question being fundamental is that in order to determine the (un-) certainty of an estimate most cost estimators will use Monte Carlo simulations applying Central Limit Theorem based probability density functions, i.e. triangular or normal, although the required (minimum) number of independent observations verifying this will not be available. Especially in respect to aerospace innovations very few if any actual observations will be available within the specific context and the analogous use of observations from other contexts, as offered through comparative databases of various software solutions, does not meet these criteria either.

A further important reason for this question being fundamental is that the type of observation commonly used is financial cost for individual work-breakdown structure elements. This stands in contrast to the state-of-art recommendations which put forward the use of a risk management process which is based on effort level scoring schemes and custom probability / likelihood ranges. While the outcome may be a financial range on effort level, the unit of measurement is based on patterns of categories of impact and probability which is fundamentally different from TBE estimation efforts.

4.3 State-of-past: The history of UQ

Drawing on Fienberg [46] the time period from approx. the mid-16th century to the present day was considered, whereby this boundary was drawn based on the assumption that the rise of probabilistic research can be seen as beginning with the work of Cardano on games of chance [48] and then Laplace on the law of large numbers. A second boundary was drawn after the work of Reverend Thomas Bayes with the Bayes Theorem and then a third boundary drawn with the growing understanding of entropy as explored by Shannon [49]. A turning point in the development of UQ metrics might also be seen in the introduction of calculable uncertainty into economic theory in the 1930s by Boy [50] and the growth of statistical approaches in industry [51], followed by the "Theory of Games and Economic Behavior" by von Neumann and Morgenstern [52] which reached a pivot point with the Nobel prize for efforts in modern portfolio theory and the capital asset pricing model in 1995. The Second World War accelerated the development of techniques, especially in the field of cryptology, followed by the growth in global trade and stock markets. From the research perspective these developments are historically fundamental although it is accepted that many different perspectives can indeed be taken. It is also important to note that developments in all sciences can seldom be identified as linear progressions with defensible key authors since publications have not been maintained with rigor over the decades and there are no doubt many thinkers and authors who have achieved significant insights and influence but have fallen out of sight. Table 6 displays a high level timeline of leading scholars and research in UQ based upon authors and sources identified during the literature research.

Foundations		State-of-past		State-of-art	
- Pacioli, F.L., (1380) “Summa de arithmetica, Geometrica, Proportiono, et Proportionalita” - Cardano, G. (mid-16th century) “The Book on Games of Chance” - Pascal, B., de Fermat (1654), P. on Fair Prices - Graunt, J. (1662) “Natural and political observations made upon the bills of mortality”	- Bernoulli, D. (1738) on utility theory - Bayes, T. (1764) on inverse probability method - Legendre (1805) on the method of least squares - Gauss (1809) on normal distribution errors and least squares - Laplace, P.S. (1810) on the central limit theorem - Quetelt, A. (1835) on concept of the average man	- Galton, F. (1885) on regression towards the mean - Galton, F. (1888) on the concept of correlation - Pearson, W. (1900) on the chi-square test - Fisher, R.A. (nd) on significance testing - Gosset, W.S (1908) on the student t-distribution - Knight, F.H. (1921) “Risk, Uncertainty and Profit“[184]	- Fisher, R.A. (1925) “Statistical methods for research workers” - Pearson, W. (1935) “The Application of Statistical Methods to Industrial Standardization and Quality Control” [51] - Neyman, J. (1934) on the confidence method - Shewhart, W. (1939) “Statistical Method from the Viewpoint of Quality Control” [179]	- U.S. Department of Defense (2006) “Risk Management Guide for DoD Acquisition“[129] - Haskins, C. ed. (2007) “INCOSE Systems Engineering Handbook v. 3.1” [15] - RAND Corporation (2007) “Evaluating Uncertainty in Cost Estimates” - U.S. Air Force (2007) “Cost Risk and Uncertainty Analysis Handbook“	- RAND Corporation (2013) “Making Good Decisions Without Predictions. Robust Decision Making for Planning Under Deep Uncertainty” [157] - United States Naval Center for Cost Analysis (2014) “Joint Agency Cost Schedule Risk and Uncertainty Hand Book” [186]

- Arbuthnot (1712) on devine providence Bernouilli, J. (1713) “Ars Conjectandi” - Bernoulli, J. (1713) on subjective probability - De Moivre, A. (1718) “The Doctrine of Chances: or, A Method of Calculating the Probability of Events in Play”	- Maxwell (1859) work on the kinetic theory of gases and law errors - Galton, F. (1869) “Hereditary Genius: An Inquiry into its Laws and Consequences”	- Neyman, J. (1923) “On the application of probability theory to agricultural experiments. Essay on principles”	- Jeffreys, H. (1939) “Theory of Probability” - Shannon, C.E. (1948) "A Mathematical Theory of Communication" [49] - Kolmogorov, A.N.(n.d.) on probability axioms	- International Society of Parametric Analysis (2008) “Parametric Estimating Handbook” - National Aeronautics and Space Administration - NASA (2008) “Cost Estimating Handbook“[10]

Table 6. Map of the leading scholars and areas of research in UQ

4.4 **Uncertainty quantification metrics identified**

Table 7 provides an overview of uncertainty quantification metrics identified in industry guides (i.e. reports, standards or technical guidelines), journal papers, conference contributions, and PhD theses. Methods for identifying or quantifying uncertainty or variables influencing the magnitude or behavior of uncertainty metrics were not considered. Particular care was taken to focus on the metrics describing data patterns and not their interpretation, i.e. a (strange) attractors or thresholds in a dataset or the concept of randomness are considered as behavior of data versus being an objective metric. This focus led not only to the identification of metrics (as defined by having a specific unit of measure), but also to the identification of metric "families" to which these metrics can be sorted.

Table 7. Exemplary UQ metrics in literature

Source	Date	Type	Discussed Metrics
Abebe, A.J., Guinot, V., et. al. [53]	2000	Conference	CDF, FS, IQR, PDF, MEM, TPE
AF CRUH [36]	2007	Guide	IQR, PDF, R^2, SD, SK
Alexander et al. [54]	2004	Report	CDF, CI, MM, PR, SD
Andersson, B.A., Bellomo, S., et. al. [55]	2013	Report	CI, PDF, RVC, TPE
Ansari, S., Bell, J., et. al. [56]	2006	Journal	PR
Arena, M.V. et. al. [57]	2006	Guide	CC, CDF, IQR, MEM, N, PDF, PR, RC, SD, TPE
Asiedu, Y., Gu, P. [65]	1998	Journal	CDF, CI, MM, PDF, TPE
Augusdinata, B. [37]	2008	Thesis	BR, BV, CDF, CI, DF, IQR, MEM, MM, PDF, PR, PV, R^2, S, SD, SK, STAT, V
Aven, T. [58]	2013	Journal	BR, BV, CI, MEM, N, PDF, SPE
Baguley, P. [59]	2004	Thesis	FS
Banazadeh, A., Jafari, M.H. [60]	2012	Journal	N, PDF, R^2, S
Bankole, O., Roy, R. et. al. [61]	2012	Journal	CI, IQR, MEM, MM, PDF, SPE, V
Bearman, N.E. [62]	2013	Thesis	ADP, CO, DH, SB, SM, T, TQ

Black, H.M. [63]	2008	Survey	CDF, MEM, PDF, SD, TPE
Celaya, Saxena et. al. [64]	2012	Conference	BR, FS, N, PDF, R^2, S
Chalupnik, M.J., Wynn, D.C., et. al. [66]	2013	Journal	BV, DF, PDF, S
Curran, R., Raghunathan, S. et. al. [67]	2004	Journal	CDF, FS, IQR, MM, PDF, PR, R^2, TPE
DeCarlo, L.T. [68]	1997	Journal	BV, DF, IQR, K, MEM, N, PDF, PV, SD, SH, SK, STAT
Dieckmann, N.F., Mauro, R. et. al. [69]	2010	Journal	CI, IQR, MEM, PDF, PR, R^2, RVC, S, SK, SPE, TPE
Durugbo, C., Erkoyuncu, J.A. et. al. [70]	2010	Journal	IQR, N, PR
Dysert, L.R. [71]	2008	Conference	R^2, STAT
Erkoyuncu, J.A, Roy, R. et al. [72]	2011	Journal	CDF, DF, FS, MM, PR, PV, S, SD
Erkoyuncu, J.A. [73]	2011	Thesis	CDF, CI, MM, PDF, R^2, SD, SPE, TPE
Erkoyuncu, J.A. Durugbo, C., Shehab, E. et al. [74]	2013	Journal	BV, CDF, CI, DF, FS, MM, PDF, R^2, S, SK, TPE
Erkoyuncu, J.A., Durugbo, C., Roy, R. [75]	2013	Journal	MM, PDF, TPE
Faller, W., Schreck, S.J. [76]	1996	Journal	NN
Ferguson, R., Goldenson, D., et. al. [77]	2011	Guide	BR, CDF, MEM, MM, MSE, PDF, PR, R^2, S, SD, SK
Fiori, A.M. [78]	2008	Journal	CDF, DF, IQR, K, MEM, N, PDF, SD, SK

Galvao, A.F., Montes-Rojas, G. et. al. [79]	2013	Journal	BV, IQR, K, N, PDF, PV, R^2, S, SD, SK
Galway, L.A. [80]	2007	Report	BR, CI, IQR, MM, PDF, PR
GAO [81]	2009	Report	BV, CI, IQR, PR, SPE
Goddard GSFC-STD-0002 [82]	2009	Guide	PR
Goh, Newnes et al. [83]	2010	Journal	CDF, FS, IQR, PDF, PR, R^2
Golkarl, A., Crawley, E.F. [4]	2014	Journal	BV, DF, IQR, MEM, MM, MSE, N, PDF, PR, R^2, RC, S, SD
Grenn, M.W., Sarkani, S., Mazzuchi, T. [84]	2014	Journal	DF, EP, PR, V
Haase, N., Renkewitz, F. et. al. [84]	2013	Journal	CI, IQR, MEM, PR, PV, RMSD, S, SD
Hallegatte, S., Shah, A. et al. [85]	2012	Report	BR, BV, DF, PDF, PR, SD
Hamarat, C., Kwakkel, J.H. et. al. [86]	2013	Journal	BV, CC, DF, IQR, MEM, PDF, TR
Haskins, C., ed. [15]	2007	Guide	BV, CI, PDF, PR, R^2, S
Hillson, D.A. [87]	2005	Conference	BV, IQR, MEM, MM, PR, SPE, TPE
Hofmann, M. [88]	2005	Journal	CC, DF, SD
ISO/IEC 15288 [89]	2008	Standard	PDF, TR
ISPA [90]	2008	Guide	DF, IQR, K, R^2, SD, STAT
Kennedy, M.C., O´Hagan, A. [91]	2001	Journal	BR, CI, MEM, PDF, R^2, RVC, S, TPE

Khodakarami, V., Abdi, A. [92]	2014	Journal	BR, BV, CDF, IQR, MEM, MM, PDF, R^2, RVC, SD
Kreye, M.E., Goh, Y.M. et. al. [93]	2012	Journal	CI, DF, FS, IQR, MEM, MM, PDF, PR, PV, SPE, STAT
Kwakkel, J.H., Auping, W.L. et al. [94]	2013	Journal	BV, CC, CDF, DF, IQR, MEM
Kwakkel, J.H., Pruyt, E. [95]	2013	Journal	DF, IQR, PDF, PV, R^2, TPE, V
Lee, S.H., Chen, W. [96]	2009	Journal	CI, DF, K, MEM, N, PDF, R^2, RVC, SD, SK
Lempert, R.J., Collins, M.T. [97]	2007	Journal	BR, BV, CI, DF, FS, IQR, MM, PDF, PR, S, SD, TC, TR
Lempert, R.J., Groves, D.G. et. al. [98]	2006	Report	BR, BV, CC, DF, PDF, TR
Mahnovski, S. [99]	2007	Thesis	BV, PDF, PR
Marion, T.J., Meyer, M.H. [100]	2011	Journal	BC, CDF, MEM, PV, R^2, SD, STAT
NASA CEH [10]	2008	Guide	CDF, CI, IQR, PDF, R^2, SD, SK
NATO RTO-TR-SAS-069 [101]	2009	Guide	CDF, DF, IQR, MEM, PDF, PR, S, SPE, TPE
Niazi, Dai et al. [102]	2006	Journal	FS
Nilchiani, R., Rifkin, S. [103]	2013	Report	BR, BV, CDF, PDF, PR, R^2
Nolan, A., Pickard, A. [104]	2008	Practice	PDF, PR
Patt, A.G., Schrag, D.P. [105]	2003	Journal	CI, IQR, PR, PV, S, SPE

Price, M., Raghunathan, S., et. al. [106]	2006	Journal	CDF, DF, IQR, PDF, PR, R^2
Rakow, T. [107]	2010	Journal	PR, RB
RAND Project Air Force [35]	2007	Guide	R^2, RMSD , STAT
Rech, J.E., Yan, R. [108]	n.d.	Guide	ACE, BV, CDF, CI, DF, IQR, MEM, MM, PDF, PR, R^2,RC, S, SD, STAT, TC, TPE
Rittel, H.W., Webber, M. [109]	1973	Journal	CC, DF, N
Rostami, J., Sepehrmanesh, M. et al. [110]	2013	Journal	BV, CDF, IQR, MM, PV, R^2
Roy, R., Sackett, P. [111]	2003	Report	BV, DF, PDF, PR, R^2, SPE
RR TRN 3152 [112]	2008	Guide	BV, CC, DF, PR, TC, V
Scales, J.A., Tenorio, L. [113]	2001	Journal	BR, IQR, MEM, MM, MSE, N, PDF, PV, R^2, SD
Smart, C.B. [114]	2012	Journal	BV, CDF, CI, CTE, K, MEM, PDF, R^2, RVC, SD, SK
Smit, M.C. [115]	2012	Journal	CI, MEM, PDF, R^2, S, TPE
Spackova, O., Sejno, J. et. al. [116]	2013	Conference	BR, CI, IQR, MEM, PDF, R^2, SD, SK
SSCAG [117]	2005	Guide	BV, CI, IQR, K, MEM, MM, PDF, R^2, SK, SPE, TPE
Tammineni, S.V., Rao, A.R. et. al. [118]	2009	Journal	CDF, CI, PDF, R^2, S, SD, TPE

Trivailo, O., Sippel, M. et. al. [119]	2012	Journal	CI, IQR, MUPE, PDF, PR, PV, R^2
Uffink, J.B.M. [40]	1990	Thesis	CDF, DF, EP, IQR, K, MEM, PDF, PR, SD, SH, SK
Wheeler, D.J. [120]	2012	Conference	AC, IQR, K, MEM, PDF, PR, PV, R^2, SD, SK
Xu, Y., Elgh, F. et al. [121]	2012	Journal	BV, FS, IQR, PDF, R^2, S, SD, STAT
Yao, W., Chen, X. [122]	2011	Journal	AD, BR, CDF, FS, IQR, K, MEM, MM, MSE, PDF, PR, S, SD, SH, SK
Yoe, C. [123]	2000	Report	BV, CDF, CI, DF, IQR, MEM, MM, N, PDF, PR, S, SK, SPE, STAT
Younossi, O., Lorell, M.A. et. al. [124]	2008	Report	PDF, PR
Zadeh, L.A. [125]	1965	Journal	AR, FS, PDF, PR

4.5 State-of-practice

The concept of aerospace innovation covers a very wide field of systems whereby a clear separation needs to be made between incremental advancements of established technologies and the leaps of innovation as explored by Allen [126]. While the case studies examined in this paper focus primarily on more significant incremental advancement, i.e. supersonic capabilities, the review did include some where fundamental research in the sciences is/was still in early stages (i.e. those based on novel physics developed by NASA for space travel).

A review of state-of-practice begins by visiting the cost estimator of today who is faced by the challenge of determining the uncertainty of an aerospace innovation related cost estimate. The estimator will face the common situation that the aerospace innovation context to be estimated might be summarised as "… harsh and non-forgiving. New programs often uncover the unknown unknowns. Early flights of a new system have often revealed problems of which the designers were unaware." [127]. Indeed the life span itself is significantly higher than products generally produced in industry, i.e. over 100 years [128]. The technical baseline estimate has already been created based on a work breakdown structure where each task has been assigned to the relevant supply chain units with the request for commitment to a single point estimate they are to provide. These single point estimates are then aggregated and a contingency added on top. This result becomes the estimated total aerospace innovation cost for planning and forecasting purposes from a business perspective. Various stage gates in the relevant whole product life cycle management process are then progressed through as the aerospace innovation rises in technical readiness level and the cost estimate may be revisited regularly. The cost estimate will change over time and this change may well be significant enough to challenge the overall initial commercial proposition. The more accurate the prediction of this change, i.e. the description of the change dynamics over time (uncertainty), the more effectively cost and commercial control mechanisms can be put in place. Important to note is that while state-of-practice techniques (i.e. use of probability density functions) may be used for elements of the work breakdown structure and while these may be revisited at regular points in the whole product life cycle, the cost estimation process typically ends at this point. While this perspective might appear to do injustice to many efforts made by cost estimators, it appears to be daily reality for most considering the time and resource constraints in place and, perhaps most importantly, the expectations of business decision makers, i.e. "give me a number to work with" as quickly as possible.

State-of-practice might best be validated through a review of case studies. An initial review of commercial, complete military, space mission (launcher) and publicly funded aerospace development efforts was therefore completed based on publicly available information.

Commercial examples with cost information examined included the Rolls-Royce RB211, the Boeing 787, the Spike Aerospace S-512, the Gulfstream 550, the Concorde, a prototype of a Japanese supersonic plane called the "Javelin", the Tupolev Tu-144, the Boeing X-48, the Northrop B-2 "flying wing", the Lockheed "box wing design" and the Boeing Business Jet (as part of the Boeing Supersonic Aircraft Programme).

Complete military aircraft examples with cost information examined included the B2-Spirit, the F-22 Raptor, the C17A Globemaster III, the P-8A Poseidon, the VH-71 Kestrel, the E-2D Advanced Hawkeye, the F-35 Lightning II, the V-22 Osprey, the EA-18G Growler and the F/A-18 Hornet. Examples examined where cost information proved unavailable, but which provided sufficient documentation for exploring cost uncertainty variables to some degree included the supersonic jet prototype being developed by the Japan Aerospace Exploration Agency, the LEAP-1x next-generation jet engines from GE, the Boeing 737MAX jets, COMAC's C919 plane and the Lockheed Martin Corp. SR-72 which is planned to enter development in demonstrator form as soon as 2018.

Space mission examples with cost estimation information examined included the NASA Shuttle, SpaceShipTwo from Virgin Galactic, the NASA Warp Project, the NASA Evolutionary Xenon Thruster (NEXT) project, the NASA Ion Propulsion Engine and the Skylon aircraft using the Scimitar engine and A2 airframe.

Finally space mission launchers with cost estimation information were examined including the Atlas 5-401 booster, the Falcon 9 rocket, and the U.S. Air Force's Minotaur rocket.

Additionally a number of well documented case studies of publicly funded efforts were reviewed, including the Long-Term Advanced Propulsion Concepts and Technologies project (LAPCAT), the Environmentally friendly high speed aircraft (HISAC) effort, the Skylon airframe, Distributed Propulsion Systems (i.e. NASA N3-X) including the four "corners" of the technical trade space as defined by the NASA

Subsonic Fixed Wing (SFW) Project, the EFE programme, the Low-carbon Engine Technology (SILOET) programme, the Environmental Lightweight Fan (ELF) programme, the Autonomous Systems Technology Related Airborne Evaluation and Assessment (ASTRAEA), the E3E programme, the Low Emissions Core-Engine Technologies (LEMCOTEC) project, the Clean Sky JTI (Joint Technology Initiative) effort, the New Aircraft Concepts Research (NACRE) programme, the NEW Aero Engine Core concepts (NEWAC) effort, the EnVIronmenTALly Friendly Aero Engine (VITAL) project, the European Low Emission Combustion Technology in Aero-Engines (ELECT-AE) programme, the integration of technologies in support of a passenger and environmentally friendly helicopter (FRIENDCOPTER) programme, the Technologies and Techniques for New Maintenance Concepts (TATEM) project, the Innovative Future Air Transport System (IFATS) effort, the HIgh Power Electric pRopulsion (HIPER) effort, and general defence aerospace innovation efforts. In all cases cost information was available only from a single point estimate perspective, although in some the degree to which these were exceeded were discussed.

4.6 State-of-art

State-of-art can best be understood through a review of industry guides, standards and reports. These represent a first level of transformation of theory to practice. State-of-art in essence differs from state-of-practice in that the cost uncertainty is not described by a fixed contingency on the single point technical baseline estimate, but through a cost risk process as discussed by the U.S. Air Force [36], the National Aeronautics and Space Administration [10], the International Society of Parametric Analysis [90], the Goddard Space Flight Center [82], INCOSE [15], the Casualty Actuarial Society [108], Rolls-Royce [112], NATO [101], the Space Systems Cost Analysis Group [117], RAND [35], and the U.S. Department of Defence [129]. The uncertainty quantification metrics for state-of-art are illustrated in Table 8.

Table 8. Frequency of uncertainty quantification metric reference in state-of-art literature review

Rank	Metrics	Frequency
1	PDF	17
2	PR	15
3	BV, IQR	10
4	R^2	9
5	CDF, CL, DF, SD	8
6	MEM, MM	6
7	BR, S, SK, SPE, TPE	5
8	STAT	4
9	CC	3
10	K, N, RC, TC, TR	2
11	ACE, ADP, CO, DH, MSE, RMSD, RVC, SM, T, TQ, V	1
12	AC, AD, AR, BC, CTE, EP, FS, MUPE, NN, PV, SH, PV	0

4.7 Comparing state-of-practice to state-of-art

A recent industry survey [63] (previously completed 10 years before) succinctly summarizes “how the U.S. Aerospace Industry (Government and contractor) develops and applies cost risk analysis to aid business decisions.” and comes to the conclusion that “Aerospace program cost overruns and schedule slides have created considerable angst, funding issues, and negative headlines. As a result, DoD and NASA increasingly emphasize the importance of cost risk management and “cost realism” (i.e., “data-driven” estimates)”. Although uncertainty quantification is becoming more and more objective, the survey respondents do note that subjective methods still dominate 60% of the time with all the issues related to expert judgment of uncertainty [130] or differing stakeholder risk perspectives [131]. From a metric perspective only standard single data center driven statistics are mentioned as being used by respondents, while the scarcity of historical data was raised by 75% of respondents as the most significant hurdle to uncertainty quantification. It is unclear whether this scarcity refers to data as a whole, or data which follows only single data center characteristics. Almost 2/3 of all cost estimations are hereby conducted in MS® Excel versus in professional cost estimation tools such as COCOMO or PRICE H.

State-of-practice is increasingly influenced by the most representative state-of-art contributions which cluster in the period of 2005-2009 and are predominantly published by U.S. governmental space and defense organizations. Key contributions here include Crawford [132], AFSC [133], SMC/FMC [134], the U.S. DoD [135, 136], Dienemann [137], the FAA [138], National Audit Office [139, 140], the Northrop Corporation [141], McNichols [142], Wallenius [143], and Weiss [144]. The metric focus is based on those associated with single modal probability density functions and the methodologies involved make a clear separation between the generation of technical baseline estimates and the ensuing cost risk process. Of particular note perhaps is that default Central Limit Theorem based probability density functions are still typically recommended as starting points and parametric techniques commonly applied.

In general this current industry practice can be considered as a response to the United States General Accounting Office's report to the Subcommittee on Space and Aeronautics, Committee on Science, House of Representatives on "Lack of Disciplined Cost-Estimating Process Undermines NASA's Ability to Effectively Manage Its Programs" [145]. This report identified major causes of cost growth including "incomplete cost risk assessment, acquisition workforce problems, corporate-directed actions, competitive environment, and flawed initial program planning" [10]. The ensuing RAND report "Improving the Cost Estimation of Space Systems Past Lessons and Future Recommendations" [124] then consolidated this into a set of recommendations that triggered first the "U.S. Air Force Cost Risk and Uncertainty Analysis Handbook" [36] and then the NASA Cost Estimation Handbook [10] including relevant efforts by the SSCAG [117]. A key recommendation of the following GAO report [81] hereby was to "Conduct a cost risk assessment that identifies the level of uncertainty inherent in the estimate" [10]. The U.S. Air Force [36] presents cost uncertainty analysis as that step in the cost estimation method which applies the “Formal Risk Assessment of System Cost Estimates” (FRISK) method [146] to identify the impact and probability of various variables on the technical baseline estimate. The technical baseline estimate is determined in advance and should not include uncertainties, but focus on determining most likely single point estimates (often using default distributions for orientation). The FRISK method then determines the uncertainty of the technical baseline estimate in order to recommend financial provisioning for such in budgeting processes. Based on the default shape of the probability density function most fitting to the overall risk profile the metrics suggested for uncertainty quantification are IQR, PDF bounds, R^2, SD and SK.

Similar to the U.S. Air Force [36] NASA [10] proposes a methodology which clearly separates between the cost estimate, called “life cycle cost” point estimate, and the cost estimate uncertainty which is determined through a cost risk determination process. In comparison to the U.S. Air Force the method is then extended to the six NASA phases of the project life cycle and the concept of cost readiness levels applied. While no specific cost risk policy is put forward guidance is recommended through the relevant NASA Policy Directives, NASA Procedural Requirements and Cost Risk Volume 2 [10]. Hereby it is NPR 8000.4 Risk Management Procedural Requirements which outlines the relevant risk management process including the calculation of risks and uncertainties. Important to remember hereby is that in contrast to the small series focus of the U.S. Air Force [36], the NASA approach is designed for application to major space flight projects where the unit of one dominates. The other factors raised by the U.S. Air Force [36] are also of relevance, although an extension is made in respect to emphasizing the need for deriving the cumulative density function itself. FRISK [146] is again put forward as the relevant risk assessment methodology. In addition several commercially available cost modeling tools are recommended including NAFCOM [147], PRICE H by Price Systems, SEER H by Galorath and COCOMO. In respect to estimation software it is also important to note that due to methodological and mathematical calculation differences results for similar calculations may differ widely [54] or be prone to generic user errors [148]. Further notable contributions in this timeframe were by Fox [149], Lillie [150], Arena [57], and RAND [151].

The NASA approach is the most stringently codified method available and is designed for the cost estimation of typically single units for a single mission or very small series (i.e. reusable launch vehicles). In respect to small series (i.e. production units of several hundred) the U.S. Air Force Cost Estimation Handbook provides solid orientation. In respect to innovative large series (i.e. production units of several thousand for commercial aircraft) a gap emerges. Commonly accepted cost estimation methodologies for pure research and development projects also do not exist. In the NASA Cost Estimation Handbook [10] "Figure 1-7. The Cost Estimating and Budgeting Connection" illustrates how single mode probability density functions are used to estimate cost ranges, whereby skew increases over the estimation process with kurtosis decreasing. Volume 2 is then specifically focused on cost risk. In section 2.2.2 the activity "Quantify Cost Estimating Uncertainty" is specifically mentioned. In this volume

NASA explicitly emphasizes the importance of "distinguishing between uncertainty (lack of knowledge or decisions regarding program definition or content) and risk (the probability of a predicted event occurring and its likely effect or impact on the program)". From a general project perspective efforts do remain relevant in respect to estimation “short-cuts” [152, 153].

Based on the NASA approach the starting point for the determination of cost estimate uncertainty is a single point estimate for the technical baseline cost [10]. The next steps are determining the co-efficient of dispersion, deriving the cumulative density function and determining confidence levels. The probability density function of the program’s total cost is hence derived from the single point estimate, the single point estimate probability, and the co-efficient of dispersion. Combining this function with the single point estimate and the confidence level then determines the “risk dollars” to be allocated as a measurement of cost estimate uncertainty. This is followed by a sensitivity analysis which enhances the determined uncertainty with factors such as the uncertainty of all cost estimating relationships and economic factors. Due to the low technical readiness level of most NASA efforts standard probability density functions are recommended (although without theoretical grounding for the recommendation) and thoroughly described including guidance under which conditions they should be used and benchmarks of relevance. Similar can be found in the SSCAG [117] and U.S. Air Force approach [36]. In the practice of estimators this available spectrum of approaches however typically reduces to the triangular distribution since it is fairly simple to characterize; the estimator only needs to produce three points: a reference point (sometimes called the “most likely”), a pessimistic point (upper boundary) and an optimistic point (lower boundary). Determination of the boundaries is then most often the result of an expert opinion elicitation process [36]. The SSCAG [117] provides similar examples and guidance on technical risk distributions while the U.S. Air Force approach [36] provides guidance and examples of selecting single modal uncertainty distribution shapes and bounds for the subjective assessment of technical input risk. All sources attempt to provide benchmark data from various programs for orientation purposes as well.

4.8 State-of-future

The state-of-future can best be understood through a review of PhD theses, conference contributions, journal articles and the work of research organizations such as listed in Table 9.

Table 9. Exemplary research organizations and research focus

Research organization	Research focus of relevance
RAND Institute	RAND Project AIR FORCE 2014 Resource Management Program weapon-system cost estimating theme (http://www.rand.org/paf.html).
Delft University of Technology	Space engineering department cost estimation in spacecraft design and analysis, and risk management (http://www.lr.tudelft.nl/en/organisation/departments/space-engineering/space-systems-engineering/expertise-areas/spacecraft-engineering/design-and-analysis/)
New England Complex Systems Institute	Engineering focus on evolutionary dynamics and distributed collaborative design (http://www.necsi.edu/research/engineering/)
Cranfield University Complex Systems Research Centre	Complex and adaptive systems including properties of emergence (http://www.som.cranfield.ac.uk/som/p1077/Research/Research-Centres/Complex-Systems-Research-Centre)
London School of Economics Complexity Research program	AAPS product definition stage, complex social systems and emergence (http://www.lse.ac.uk/researchandexpertise/units/complexity/home.aspx)

Key concepts of relevance for the state-of-future include; entropy, complex adaptive systems, uncertainty threshold responses, and deep uncertainty whereby the general understanding is that pattern recognition approaches need to take precedence over pattern matching normalisation to probability density functions based on the Central Limit Theorem. While a plethora of research efforts have been undertaken in these fields the below activities appear most suited for supporting the aerospace innovation cost uncertainty quantification focus of this review.

- In the conceptual area of entropy efforts in general build on the work of Shannon [49] in information theory with a special focus on information transmission, whereby Zurek [154] expands this solidly into reflections on algorithmic randomness, Uffink [40] hardens the mathematical underpinnings and linkages to physics, and Grenn et. al. [84] make first attempts to transfer the entropy principles into the systems engineering space.
- In the conceptual area of complex adaptive systems the most notable efforts appear to be around the concepts of complex adaptive systems engineering [155] where especially human factors and collaboration influences gain prominence in seeking to understand overall complex engineering efforts. This then maps closely with reflections concerning the manner in which engineering environments develop from chaotic, through complex and complicated to the simpler structures found in industrial series manufacturing [156].

- In the conceptual area of uncertain threshold response the emphasis remains similar to adaptive robust design approaches where the basic perceptions of risk levels in scenarios and robust versus optimal approaches are discussed [97, 157, 158]. There are at the same time links here to the questions of scenario management and system dynamics especially as related to deep uncertainty as discussed below. At the same time various related concepts can be included here such as uncertainty propagation methods [96] and the Bayesian calibration of computer models [91, 159, 160, 161].
- In the conceptual area of deep uncertainty the fields of general policy analysis from the perspective of adaptive robust design [86] and dynamic scenario discovery [94] form current areas of especially relevant research in addition to the further developments from the perspective of exploratory modelling and analysis [94, 162, 163, 165, 166, 167, 168, 169, 170, 171].

Underlying the research study exploration is hence the assumption that the sustainable performance of complex process landscapes, such as the whole product life cycle, depends on their ability to reconfigure themselves in the same emergent manner as demonstrated by complex adaptive systems under changing environmental conditions. Enabling process landscapes to reconfigure themselves in an intelligent manner is an emerging effort in many industries and is only slowly becoming feasible as the relevant automation and industrialization technologies become increasingly available. The uncertainty quantification metrics put forward for state-of-art are illustrated, ranked by frequency, in Table 10.

Table 10. Frequency of uncertainty quantification metric reference in state-of-future literature review

Rank	Metrics	Frequency
1	PDF	30
2	IQR	23
3	R^2	20
4	MEM	18
5	PR	17
6	SD, DF	16
7	CL, S	15
8	CDF	14
9	BV, MM	13
10	PV	11
12	FS, N	10
13	TPE	9
14	SK	8
15	BR, K	6
16	RVC, SPE	5
17	CC, STAT	4
18	MSE, V	3
19	SH, TR	2
20	AD, AR, BC, CTE, EP, MUPE, NN, RC, RMSD, TC	1
21	AC, ACE, ADP, CO, DH, SM, T, TQ	0

While certain conceptual areas can be identified we must also differentiate between the metric of choice and the method chosen for its presentation. Besides traditional approaches for statistical visualisation interest does appear to be rising in respect to using alternative approaches such as sound or augmented reality for representation of such complex data. Such are illustrated in Table 11.

Table 11. Frequency of uncertainty quantification metric reference in PhD theses

Rank	Metrics	Frequency
1	PDF	4
2	CDF, PR, SD	3
3	BV, CL, DF, IQR, MEM, MM, R^2, SK	2
4	BR, EP, FS, K, PV, S, SH, SPE, STAT, TPE, V	1
5	AC, ACE, AD, ADP, AR, BC, CC, CO, CTE, DH, MUPE, MSE, N, NN, RC, RMSD, RVC, SM, T, TC, TR, TQ	0

5 Critical analysis

Summary of this section:

1. *Uncertainty quantification is an emergent field of research.*
2. *The frequency of uncertainty quantification metrics mentioned changes across the states.*
3. *The time from theory to mainstream adoption spans human and product generations.*
4. *Literature does not differentiate between static and dynamics uncertainty quantification.*
5. *An integrated baseline uncertainty quantification typology is required.*
6. *Hindsight, insight and foresight are key perspectives in practice.*
7. *Multiple plausible future scenarios challenge metric selection.*

The literature research results shared in this paper provide an overview of metrics used to quantify the cost risk uncertainty of technical baseline estimates prepared for aerospace innovations. The results are structured based on the state-of-past (which metrics used to be used), the state-of-practice (which metrics are currently used), the state-of-art (which metrics are available for use) and the state-of-future (which metrics could be used in the future). This structure was chosen to emphasize that uncertainty quantification is an evolving field of research which finds its origins in human psychology, religion and the social sciences, followed the evolution of scientific thinking and industrialization, and is now maturing to a more differentiated view of quantifying and predicating the future. To a degree we may be witness to the end of a revolution which started with SoS as the unit of analysis or interpretation, learned to disaggregate the SoS into its components based on the assumption that this would allow for better understanding of the SoS, to then slowly recognize that in an increasingly networked and interdependent world this SoS is having greater and greater impact on the future so that it requires revisiting.

5.1 Rank changes

The first step completed in the critical analysis was to review the changes in uncertainty quantification metrics across the states, based on the frequency these were identified in the reviewed literature. The state-of-art was hereby focused to consider industry guides, reports and standards, while the state-of-future was focused on journal contributions. Theses contributions were considered separately since these do not necessarily represent contributions to knowledge which are accepted as more reliable by the peer community than the other sources, i.e. they are typically precursors to further journal articles and then, with growing acceptance these concepts may become more tangible in the form of state-of-art literature sources. uncertainty quantification metrics in total were identified and ranked as illustrated by Table 12.

Table 12. Uncertainty quantification metric rank changes

All			State-of-art			State-of-future				Theses		
Metric	Freq.	Rank	Metric	Freq.	Rank	Metric	Freq.	Rank	Change	Metric	Freq.	Rank
AC	1	22	AC	0	N/A	AC	N/A	20	GONE	AC	0	5
ACE	1	22	AD	0	N/A	ACE	N/A	20	GONE	ACE	0	5
AD	1	22	AR	0	N/A	ADP	N/A	20	GONE	AD	0	5
ADP	1	22	BC	0	N/A	CO	N/A	20	GONE	ADP	0	5
AR	1	22	CTE	0	N/A	DH	N/A	20	GONE	AR	0	5
BC	1	22	EP	0	N/A	SM	N/A	20	GONE	BC	0	5
CO	1	22	FS	0	N/A	T	N/A	20	GONE	CC	0	5
CTE	1	22	MUPE	0	N/A	TQ	N/A	20	GONE	CO	0	5
DH	1	22	NN	0	N/A	AD	1	19	NEW	CTE	0	5
MUPE	1	22	PV	0	N/A	AR	1	19	NEW	DH	0	5
NN	1	22	SH	0	N/A	BC	1	19	NEW	MUPE	0	5
SM	1	22	ACE	1	11	CTE	1	19	NEW	MSE	0	5
T	1	22	ADP	1	11	EP	1	19	NEW	N	0	5
TQ	1	22	CO	1	11	MUPE	1	19	NEW	NN	0	5
EP	2	21	DH	1	11	NN	1	19	NEW	RC	0	5
RMSD	2	21	MSE	1	11	RC	1	19	-9	RMSD	0	5
RC	3	20	RMSD	1	11	RMSD	1	19	0	RVC	0	5
SH	3	20	RVC	1	11	TC	1	19	-9	SM	0	5

TC	3	20	SM	1	11	SH	2	18	-6	T	0	5
MSE	4	19	T	1	11	TR	2	18	-8	TC	0	5
TR	4	19	TQ	1	11	MSE	3	17	-6	TR	0	5
V	5	18	V	1	11	V	3	17	-6	TQ	0	5
RVC	6	17	K	2	10	CC	4	16	-7	BR	1	4
CC	7	16	N	2	10	STAT	4	16	-8	EP	1	4
K	10	15	RC	2	10	RVC	5	15	-4	FS	1	4
STAT	10	15	TC	2	10	SPE	5	15	-8	K	1	4
SPE	12	14	TR	2	10	BR	6	14	-7	PV	1	4
FS	13	13	CC	3	9	K	6	14	-4	S	1	4
N	13	13	STAT	4	8	SK	8	13	-6	SH	1	4
PV	13	13	BR	5	7	TPE	9	12	-5	SPE	1	4
BR	14	12	S	5	7	FS	10	NEW	1	STAT	1	4
SK	17	11	SK	5	7	N	10	11	-1	TPE	1	4
TPE	18	10	SPE	5	7	PV	11	10	2	V	1	4
MM	22	9	TPE	5	7	BV	13	9	-6	BV	2	3
S	22	9	MEM	6	6	MM	13	9	-3	CI	2	3
BV	26	8	MM	6	6	CDF	14	8	-3	DF	2	3
CI	26	8	CDF	8	5	CI	15	7	-2	IQR	2	3
DF	26	8	CI	8	5	S	15	7	0	MEM	2	3
CDF	27	7	DF	8	5	DF	16	6	-1	MM	2	3

SD	30	6	SD	8	5	SD	16	6	-1	R2	2	3
MEM	31	5	R2	9	4	PR	17	5	-3	SK	2	3
R2	35	4	BV	10	3	MEM	18	4	2	CDF	3	2
PR	38	3	IQR	10	3	R2	20	3	1	PR	3	2
IQR	39	2	PR	15	2	IQR	23	2	1	SD	3	2
PDF	57	1	PDF	17	1	PDF	30	1	0	PDF	4	1

In the state-of-future the uncertainty quantification metrics AD, AR, BC, CTE, EP, FS, MUPE, NN, PV, RB, and SH appear for the first time, with especially FS (10 mentions) and PV (11 mentions) becoming relatively frequent UQ metrics of discussion. The uncertainty quantification metrics ACE, ADP, CO, DH, SB, SM, T and TQ are not mentioned anymore, although in state-of-art these were only mentioned once each, i.e. may be of peripheral nature. While the state-of-future had 20 identifiable frequency rankings, the state-of-art only had 11 such identifiable frequency rankings indicating a smaller spectrum of uncertainty quantification metrics being discussed. Per se this can be considered as normal since the more mainstream uncertainty quantification metrics become, the more focused we can expect their usage to be because state-of-art is driven by practical considerations where too much choice does not contribute to general shared understanding. The most significant rank drops from state-of-art to state-of-future are RC (-9), TR (-8), STAT (-8), and SPE (-8). The top two ranked metrics in state-of-future remain the same as in the state-of-art, i.e. PDF (rank 1), IQR (rank 2), while PR drops to rank 5 in the state-of-future, BC drops to rank 9 and R^2 rises to rank 3.

As can therefore be expected, across the states, uncertainty quantification metrics will arise in research (i.e. PhD theses), mature towards journal contributions (state-of-future) and then industry guides, reports and standards (state-of-art) to then either become firmly embedded in industrial practice, while others will struggle to achieve adoption and become replaced by other uncertainty quantification metrics. Due to the long life cycle of aerospace solutions we can safely assume that there will be little churn in leading uncertainty quantification metrics, while, as life cycle experience grows, others may become more and more viable in specific, versus generic, estimation approaches.

5.2 Static versus dynamic

Revisiting earlier questions considering the static and dynamic nature of uncertainty quantification metrics the question also arises which metrics may be more suitable than others for representing dynamic changes in uncertainty. In this respect the approach used most often is to discuss the changes of metrics used for static uncertainty quantification over time. Unfortunately no specific literature resource could be identified in this respect and hence the question must be relegated to suggested further work.

5.3 Perspectives on states

The state-of-past is considered to be that industrial usage of uncertainty metrics since the advent of the industrial age in the early 1900s. Although significant literature exists in respect to evolution of statistical methodologies and metrics before this time and indeed one could trace the uncertainty quantification discussion back to at least the early Greek philosophers the fundamental schism of interest in this research overall is the phase change from mass manufacturing where Central Limit Theorem principles can be applied with relative confidence, to an economy where rapidly growing global interdependence, information, knowledge and innovation are driving low volume production of highly innovative products with short life cycles.

The state-of-practice is considered to be that industry usage of uncertainty metrics since 2000 as put forward in industry guides. It may best be described through the facts that over 96% of over 26 000 enterprise risk entries of an aerospace manufacturing company over a 10 year period used single point distributions when describing cost risk with the next most used distribution being a triangular one with about 3.5%. At the same it must be questioned whether a dedicated cost risk process for uncertainty quantification as advocated among others by NASA [10] is being applied at all in industry since in practice the boundary to the technical baseline estimate creation process is blurred at best. The state-of-practice in respect to uncertainty quantification can hence be summarized as being the addition of a single figure (typically called "contingency") to a technical baseline estimate, whereby the metric is a single point estimate in financial figures, a contingency in % and financial figures, and a final single point estimate in financial figures. The review of case studies mirrored this perspective in that virtually only single point estimates could be identified. The phase change from the state-of-past is not yet in full swing especially since the education of the workforce is still heavily influenced by industrial paradigms and a heterogeneous industrial landscape.

When reviewing the state-of-art the focus lies on the same time window as state-of-practice with an emphasis on journal and conference contributions. Two points of interest arise. First of all the preferred metric for uncertainty quantification is the probability density function whereby the single point probability density function is considered separately and discussed less frequently. The second point is the clear separation of uncertainty quantification from the technical baseline estimate. The low adoption of these two points, as evidenced by the state-of-art discussion, points us to the previously mentioned phase change in paradigms being well underway.

The state-of-future can best be understood by examining PhD theses since 2000 and current research activities in various relevant research institutes. Although some theses are focused on the aerospace industry a more general perspective can be taken in that across industries (i.e. water resource management or tunnel building) various uncertainty quantification metrics are being explored with the question of whether they may be more suitable to forecasting long term uncertainty. While continued investigation of the probability density function as seen from a Central Limit Theorem perspective remains an integral element, the general trend appears to be towards understanding at which point such paradigms are no longer defensible and beyond that point the suitability of approaches such as fuzzy theory [12, 172, 173, 185], Bayesian belief networks [91, 92, 159, 160, 161], and the concepts of entropy [84, 154], complexity [60, 88, 144, 156] and tail-weight [174]. An underlying theme that emerges at the same time is that of the availability of computing resources for resolving questions in polynomial time and the parallel shift to robust design under deep uncertainty.

The state-of-practice primarily reflects metrics developed in the state-of-past and reflects regression decision of estimators to the fundamental questions raised in the introduction. In state-of-practice the estimator, out of tradition and without theoretical guidance, typically choses cost information based on work-breakdown structures and standard dispersion metrics based on subjectively chosen most fitting default single center probability density functions whereby these are most likely to be of normal, triangular or log-normal nature. Commonly found metrics in state-of-practice are CL, IQR, MEM, MM, R^2, and SD.

The state-of-art points to a slowly arising paradigm shift in that the estimator, accepting the difference between cost and cost risk estimation, will chose information based on risk assessments (i.e. probability and impact) with metrics based on custom probability density functions which accept multiple data centers. Commonly found metrics in state-of-art are CC, K and SK.

The state-of-future invites the estimator to progress in that while the information source remains cost risk focused, the concept of probability density functions is abandoned in favor of multi-dimensional response surfaces that change over time. Commonly found metrics in state-of-future are related to homogeneity, density, compression, and complexity.

5.4 Framework of reflection

In the specific context of uncertainty quantification in cost estimation for aerospace innovation the "states" provide a framework of reflection of cost estimation paradigms. The state-of-practice indicates that the primary orientation given to the estimators stems from industry guides, company guidelines or from the techniques embedded in cost estimation software being used. State-of-practice serves as a framework for guiding the work of the estimator. In this respect, as mentioned previously, it is the generation of a single point estimate with a high level of confidence which is the goal. Per se we are seeing a deterministic paradigm in practice which, in highly industrialized contexts, serves the organization well since Central Limit Theorem applicability can be accepted. The less industrial the context however, the less the deterministic paradigm can confidently be accepted as being sufficient. These confidence influencers have several characteristics related to computational constraints, normalizing to Central Limit Theorem based probability density functions, multiple plausible futures, set based topology and metric topology.

- Computational restraints: significant efforts are made to increase the reliability of the single point estimate through more and more rigorous engineering break-down cost estimation approaches, the assumption being that the more accurately we can describe what we are building and how, the more accurately we can estimate the cost, or at least identify the key cost estimating relationships to open the path to probabilistic parametric approaches. The development and deployment of such efforts into operational contexts is however significantly constrained by generally available computational resources and the inherent complexity of designing cost simulation models that not only cover individual components, but the iterative aggregation of these into (sub-) assemblies, propulsion systems, airframes, mission paths, etc., as a whole. Important to note in this respect is that the more complex a cost simulation becomes the less the relevant simulation details and dynamics can be effectively communicated with stakeholders which then itself impacts overall confidence in the results [176]. Indeed it might be also be argued that the more information is available, the lower the ability to recognize patterns due to computational restraints [12].

- Normalizing to the Central Limit Theorem: a second characteristic is the increasing acceptance of basic probabilistic approaches in the use of probability density functions as discussed by the selection of most fitting probability density function where a decision-tree centered on the continuity of the data being is used so that the estimator, in the end, choses from a range of probability density functions. The focus lies on finding the most fitting default probability density function to which the data can be normalized to. This approach can help the estimator make the relevant choice of probability density function to normalize to, however the branching criteria are not given objective thresholds, i.e. while the question “Is the estimate symmetric?” is posed, no guidance is given regarding what is meant by “symmetric”, how it can/should be measured and what specific values would indicate a symmetric estimate [177]. The same applies to questions concerning confidence level or skew. Wheeler [120] builds on Shewart [178, 179] in that the starting point for selecting the most suitable probability density function is the question of data homogeneity. This is used as a starting point for exploring the suitability of diverse metrics to point to relevant default probability density functions. The role of the Central Limit Theorem as put forward by Laplace in 1810 is also critically examined. Kurtosis and skew squared then become guiding criteria for separating between mound- U- and J-shaped distributions. A threshold for the applicability of default probability density functions is suggested through definition of an “impossible region” where high skew squared values meet low kurtosis values. From the perspective of the researchers the most encouraging element of the perspectives raised is that normalization may be considered unnecessary, indeed results threatening. Important to note as well is that Wheeler [120] emphasizes the value of analysis approaches being the identification of changes, i.e. from a dynamic perspective, versus the more static “snapshot” of uncertainty statistics typically encountered. Almost 100 years apart, Wheeler [120] and Shewart [178] can both be considered as state-of-art thinkers. The goal remains the development of a single point estimate.

- Multiple plausible futures: The third, emerging, characteristic sees the estimation method less as an alternative to the previously raised characteristics, but extends these to encompass multiple plausible future scenarios, both from an engineering perspective in the sense of trade-off analyses and also in respect to varying contextual conditions such as developments in the market, the economy or legislature. Underlying this characteristic are developments in computational capability that allow for pattern recognition approaches in big data situations while at the same time making newer techniques, such as fuzzy thinking [12, 53, 59, 125, 172, 173, 180, 181] available in order to make sense of that data. While this perspective has matured to state-of-practice in general policy analysis [86, 95, 162, 163, 165, 166, 167] and does find its place in systems engineering contexts in the form of trade-off analyses, the challenges of linking this trade-off analysis with relevant cost simulation from an engineering break-down perspective remain formidable. While parametric analysis promises “good enough” techniques, resistance to such generalizations in the engineering communities that are focused on high level of detail and exactness are often significant.

- Set based typology: A fifth characteristic is related to the typology of uncertainty concepts in their own right, i.e. how to categorize these different types of uncertainty and their interrelationships. While various typologies for interpreting uncertainty quantification have been proposed [2, 182, 183] the context and literature review suggest that the interlocking dimensions of hindsight, insight and foresight are best suited for dynamic long-term contexts and then also best admit the application of mathematical set theory which suggests that the dimensions (and their interaction) may be described by an integrated general framework. Reflecting on the characteristics mentioned above there appears hence to be less of a fundamental discourse regarding the “best” approach to uncertainty quantification in aerospace cost estimation, and more the slow emergence of a process for inferring a coherent set of measures starting with basic big data understanding, through pattern recognition and various different metrics as the relevant information becomes more and more visible and understood. This might then be generalized towards an uncertainty quantification typology as illustrated by the Venn diagram in Figure 5. It is these sets (and sub-sets) which can then be considered as dimensions relevant for uncertainty quantification.

Figure 5: Uncertainty quantification typology

The set “hindsight” contains uncertainty quantification metrics which admit the Central Limit Theorem. Examples of metric families belonging to this set are point, range, and shape. The set “insight” contains uncertainty quantification metrics describing the state of estimation parameters at the time of estimate and which are expected to change before the estimate can be verified. Examples of metric families belonging to this set are complexity, compression and homogeneity. The set “foresight” contains uncertainty quantification metrics defining the time-window of the estimate and the plausible future scenarios which is of particular importance since it contains the boundary definitions for the propagation of uncertainty in the estimate. Examples of metric families belonging to this set are the chosen time intervals, the number of time intervals the estimate looks into the future, plausible boundaries and information volatility based on technical and cost readiness. Generic uncertainty quantification metrics are relevant for all sets, while the metric family “Other” could not be clearly aligned. A mathematically coherent uncertainty quantification estimate will contain only uncertainty quantification metrics which are a subset of all three sets at the time of the estimate.

- Metric taxonomy: The sixth characteristic refers to the metrics identified in the literature review as aggregated into the taxonomy described in Figure 6.

Figure 6: Uncertainty quantification metric taxonomy

For this purpose the concept of metric families is used in respect to general areas of metrics which exhibit conceptual closeness to clusters of principles. The basic clusters of principles which are deemed relevant relate to point and range estimates, SH, and information HG, CR and CM. Several metrics could not be specifically associated with these clusters however (metric family “Other”), while certain metrics were also identified as being generically relevant across a number of principle clusters.

6 Research gap

Summary of this section:

1. Cost estimators have little guidance grounded in theory for uncertainty quantification in the aerospace innovation whole product life cycle.
2. Probability fields are shaped by data volatility and the time-window for the estimate.
3. Probability fields are categorized based on information density, valence and complexity.
4. Metric fidelity is driven by technical readiness level, cost readiness level and forecast window.
5. Deep uncertainty principles are required for robust and flexible estimates.
6. Phase changes between probability fields are poorly understood.

The estimator of today has little guidance grounded in theory when it comes to the choice of the most suitable uncertainty quantification metric to predict the uncertainty of the technical baseline estimate. This leads to the assumption that the Central Limit Theorem is applicable and default probability density functions which are commonly used in the peer community are chosen. Software based cost estimation tools also put these state-of-practice choices in the forefront.

6.1 Uncertainty quantification framework

The literature review suggests that multiple uncertainty quantification metrics are available from various theoretical backgrounds and that their suitability is based on the degree that these are able to recognize a pattern in the available information which can then be propagated defensibly over the required time-frame. Foresight determines the most relevant uncertainty quantification metric, therefore (a) the time-frame for the estimate, i.e. the number of whole product life cycle phases covered before validation occurs, and (b) the volatility of the information available for pattern recognition, i.e. the technical and cost readiness levels at the time of estimate. Since both factors change over time the uncertainty quantification metrics available for choice should also be mathematically coherent and offer clear thresholds for attraction to admit iterative maturation of the uncertainty quantification estimate. The estimator may also be able to use such a framework for understanding the requirements for the next most exact uncertainty quantification metric and working to meet those as the uncertainty quantification estimate matures towards the point where it can be validated. Figure 7 illustrates these probability fields from a framework perspective. The confidence in the uncertainty quantification is highest at the bottom left where it is measured by a single point estimate and lowest at the top right where complexity metrics find application. The estimator should typically start at the top right and work to progress their estimate down to the bottom left in order to continuously improve cost readiness levels.

Figure 7: Uncertainty quantification probability field framework

Figure 7 highlights two fundamental dimensions of uncertainty quantification which frequently are raised as important influencers of confidence in cost estimates. For one the further into the future an estimate is intended to be valid for, the more we must assume that the data being used to propagate will be subject to volatility in quality, content and density. Hence we can safely assume that the original data quality will decay in relevant density over time. The time intervals in the whole product life cycle of aerospace innovations are defined by models such as ISO, CADMID or NASA technical readiness levels. Especially the phase changes are hereby of interest since that is where a significant amount of uncertainty is injected due to changes in methods, tools, techniques and reference data. The timeline of Figure 7 focuses on the number of whole product life cycle phases the estimate is intended to cover whereby the “number” is intended to describe the number of phase changes of relevance. In general the probability field clusters might best be described from the perspectives of deterministic / bivalent (A), probabilistic / bivalent (B), probabilistic / multivalent (C), fuzzy / multivalent (D), complex / multivalent (E), or chaotic multivalent (F), whereby the attributes for the boundaries between these remain unclear at this point in time.

6.2 Uncertainty quantification metrics

The degree of expected change (volatility) is subjectively determined from expert opinion and the quality of the assessment depends to a great degree on how well detailed the relevant attributes are. Assuming that the required data is a risk or stage gate register, the individual line items can be assessed or aggregated profiles from a higher perspective utilized. Metric families of relevance for similar clusters can therefore be redefined as follows:

- Complexity (CM): At the point of highest volatility and longest predication time-frame the uncertainty quantification metric family of complexity appears most suitable. Within this metric family the metric degrees of freedom appears most suitable for uncertainty quantification. In this situation the number of relevant variables affecting the uncertainty is determined, including their range of potential values. Then the maximum number of potential combinations is calculated and this factor applied to the technical baseline estimate to determine the probability field. The maximum number of combinations may be reduced through the development of more exact variable relationships based on analogy. While large ranges emerge it must be remembered that these cover the estimate across (almost) all whole product life cycle phases. According to Price et. al. [106] "typical airframe load models have approximately 200,000 degrees of freedom." from a technical baseline estimate perspective whereby these are reduced primarily by deciding which degrees of freedom are “locked” and subject to formal change management, which degrees of freedom are linked to plausible future scenarios (and subjected to formal change management) and which are purposefully considered out-of-scope. The previous metric family hereby defines the boundaries of relevant information evaluated. *Example: For a cost estimate covering the complete whole product life cycle of an aerospace innovation, 10 variables with high volatility might be identified as relevant through expert consultation. Each variable has 10 potential values / scenarios. Degrees of freedom hence equal* 10^{10} *which means that 10.000.000.000 potential future cost scenarios exist. Each scenario must then be simulated to determine the upper and lower bounds of the estimate. The size of this range describes the uncertainty of the estimate. The use of Central Limit Theorem based probability density functions for interpretation of the aggregated simulation results, i.e. to determine a "most likely value", should be avoided unless sufficiently large numbers of real examples are available for each simulated scenario. Typically helpful techniques in practice include parametric estimating, process simulation or system dynamics modeling. High performance computing requirements quickly emerge and the effort is mainly valuable to begin identifying scenarios relevant for target costing approaches moving forward.*

- Compression (CR): As more information is gathered about plausible future scenarios, variables affecting the uncertainty of the technical baseline estimate and the relevant project along the life cycle, a point is reached where the compression family of metrics becomes usable to generate more accurate uncertainty quantification than the complexity approach. The most suitable metric in this family appears to be entropy. The previous metric family hereby defines the boundaries of relevant information evaluated.

*Example: Based on the identification of plausible future scenarios using degrees of freedom and target costing approaches, each scenario is subject to a risk assessment process and the risk probability impact data is used to calculate the (Shannon) entropy of that data in order to determine the uncertainty range. The estimate is still assumed to be relevant for the whole product life cycle yet data volatility drops significantly due to the plausibility filter applied to the degrees of freedom results and the use of new data generated by the risk assessment process. The calculation of (Shannon) entropy can be completed using various MS® Excel based templates commonly available. (Shannon) Entropy is measured by a diversity score, i.e. 2.5, which is used to describe the extent that the technical baseline estimate can be expected to be exceeded, i.e. technical baseline estimate * 2.5 as defining the upper boundary. The uncertainty range can be expected to drop significantly compared to the degrees of freedom perspective which was used to determine plausible scenarios from a target costing perspective in the first place. Again the use of Central Limit Theorem based probability density functions for interpretation of the aggregated simulation results, i.e. to determine a "most likely value", should be avoided unless sufficiently large numbers of real examples are available for each simulated scenario.*

- Homogeneity (HG): The next level of the volatility / time-frame probability fields marks a transition to the homogeneity family of uncertainty quantification metrics, whereby the quantification approach shifts to fuzzy sets [172, 173, 180, 181, 185]. In essence the fuzzy set method of clustering the degree to which a data point belongs to a cluster is determined whereby the output is the number of clusters (single figure) and average degree of membership for data to each cluster (single figure per cluster). It is particularly at this level that the first (classical) probability density function patterns emerge although they are typically multi-model/cluster relationships that are not normalized to achieve state-of-practice single modal or linear relationships. The previous metric family hereby defines the boundaries of relevant information evaluated.

  Example: While degrees of freedom was used to help define a set of plausible future scenarios and the results further refined based on risk assessment with an entropy approach, the homogeneity perspective is used to begin defining uncertainty boundaries more rigorously. Using fuzzy set techniques boundaries of the previously identified clusters are calculated to determine the degree of multi-valency / membership evident. This multi-valency can be described using single modal probability density functions and inter-quartile ranges as the basis for determining the uncertainty range of the relevant scenario technical baseline estimates. Again the use of Central Limit Theorem based probability density functions for interpretation of the aggregated simulation results, i.e. to determine a "most likely value", should be *avoided unless sufficiently large numbers of real examples are available for each simulated scenario.*

- Shape (SH): Shape is based on a custom probability density function generated from the available information and returns the uncertainty as "shape" and "scale" deviation from a separately chosen default probability density function. The deviation of the custom probability density function from the "normal" distribution values in % is then transferred to the three point estimate. The previous metric family hereby defines the boundaries of relevant information evaluated. The primary challenge encountered is limitations of standard statistical software packages which quickly reach performance limits due to complexity challenges of the computations.
Example: Within the shape family of metrics the concept of "goodness of fit" sets the foundation for a first transition to the use of Central Limit Theorem based probability density functions as part of Monte Carlo like range determinations. The difference to the range family lies in the use of the deviation from normal for kurtosis and/or skew to add uncertainty to the range estimate generated. In a simplified form the (inverse) Anderson Darling score might be used as a corrective factor to the range created by the Monte Carlo simulation. A custom probability density function is used within the Monte Carlo experiments.

- Range: The range uses the same approach as the single point estimate, but returns the complete range of uncertainty calculated by the Monte

Carlo simulation. The difference to the single point estimate is that here a cumulative density function is used to indicate the uncertainty at various confidence levels and the confidence level chosen subjectively determines the single point estimate "+" a certain % in order to raise the confidence level to 100%. The previous metric family hereby defines the boundaries of relevant information evaluated.

Example: At the stage where range becomes a relevant metric family, data volatility in relation to the estimate time-window has potentially dropped to a degree where Central Limit Theorem based estimations are admissible enough. Based on the Monte Carlo technique a most likely value, best and worst case values are determined, a default Central Limit Theorem normal or triangular distribution selected and the relevant range with confidence levels is calculated. Use of normal probability density functions precludes the consideration of shape metrics such as kurtosis or skew for further definitions.

- Point (SPE): The single point estimate assigns a single uncertainty value to the technical baseline estimate, i.e. 5% and is based on the use of a Monte Carlo simulation using the technical baseline estimate as the best case and expert opinion for determining the most likely and worst values along with a default probability density function chosen such as a normal or triangular distribution. This is suitable in areas of low information volatility and when estimating within a single life-cycle phase. The most likely result of the Monte Carlo simulation output is used as the single point uncertainty estimate. The single point estimate may have a small default contingency added by decision makers or industry practice. The previous metric family hereby defines the boundaries of relevant information evaluated.

Example: The single point estimate is the typically declared preferred form of the cost estimate from the perspective of business stakeholders. The single point estimate is a single financial figure with a usually subjective low contingency added.

While the literature review suggested that the variously indicated metrics are most suitable for varying levels (or rather cluster ranges) of information density, these metrics also appear to have varying suitability for the description / containment of multiple plausible future scenarios, i.e. the deep uncertainty paradigm, whereby this again may help the estimator argue against progressing to a next threshold as long as the number of such scenarios are not reduced in and of themselves. We could argue that the further the estimation context moves to the top right the more plausible future scenarios are guarded against.

7 Conclusions and future work

Summary of this section:

1. *State-of-practice approaches are predominantly state-of-past.*
2. *Current paradigms remain deterministic.*
3. *Different uncertainty quantification metric families are relevant for different probability fields.*
4. *Pragmatic rules for uncertainty quantification metric selection and*

calculation are needed.

5. *Efficient techniques for threshold determination between metric families are required.*
6. *A fundamental paradigm shift from deterministic to probabilistic thinking is required.*
7. *Emergence needs to be recognized as a key attribute of cost estimates.*

It is believed that current approaches to whole product life cycle uncertainty calculation / estimation are struggling to produce accurate and objective results because, disregarding the multi-modal context of the object of analysis being estimated, they…

- … focus on metrics of central tendency (i.e. MEM) and measures of dispersion (i.e. SD) which find their origins in traditional utility analyses that emphasizes the value of optimal versus sub-optimal solutions based on the law of large numbers,
- … assume a static single versus dynamic multiple plausible future scenarios, and
- … assume predictable versus emergent contexts.

It is especially the mental models associated with traditional utility analysis that assume the validity of historical propagation for future projection which obscure the influence of changing context for aerospace innovations – whole product life cycles here happen neither within simple nor complicated contexts, but in complex if not on the threshold to chaotic ones [156]. The research gap in essence condenses this challenge down to the realization that industrial tools and techniques are being applied to contexts where their preconditions for use are not being met.

The paper has hence offered an integrated perspective on the key questions related to forecast ranges, metrics of relevance, volatility of data and required confidence levels as illustrated by Figure 8.

Figure 8: Integrating uncertainty quantification perspectives

The dynamic emergent nature of the future is nothing unknown to past thinkers and authors. The much quoted economist Frank Knight [184] wrote about the concept of uncertainty:

"It is a world of change in which we live, and a world of uncertainty. We live only by knowing something about the future; while the problems of life, or of conduct at least, arise from the fact that we know so little. This is true of business as of other spheres of activity. The essence of the situation is action according to opinion, of greater or less foundation and value, neither entire ignorance nor complete and perfect information, but partial knowledge."

Lempert and Popper et. al. [166] phrase this as "Deep uncertainty exists when analysts do not know, or the parties to a decision cannot agree on, (1) the appropriate models to describe the interactions (2) the probability distributions to represent uncertainty about key variables and parameters in the models, and/or (3) how to value the desirability of alternative outcomes."

A number of different terms are used for concepts similar to what we define as deep uncertainty. Knight for example contrasted risk and uncertainty, using the latter to denote unknown factors poorly described by quantifiable probabilities. These then can be considered as conditions of ambiguity where the axioms of standard probabilistic decision theory need not hold.

The general evolution of uncertainty quantification metrics follows that sequence seen in respect to most human knowledge; starting from fundamental research in the sciences through adoption in engineering practice to generalization into other sciences. The long timeframes required for this dispersion of knowledge may hereby be related to the differing scientific rigor in various fields.

Based on the research gap identified and the conclusions drawn future work is recommended in respect to rules for (dynamic) uncertainty quantification metric selection based upon a framework which provides a set of straight-forward heuristic rules for the cost estimator based on comprehensive experimental evaluation which act as an adaptive filter for choosing the most suitable metrics as the information density changes. Whether clear defined threshold (bi-furcation) values for transition boundary conditions are identifiable is a related perspective which may also support reflections on pattern identification. At the same time the series of metrics to be recommended should be mathematically congruent so that methods applied to lower information densities can also be applied to those with higher densities as desired. The metrics suggested should be suited for describing the propagation of uncertainty in the most accurate manner and not focus on the description of time slices only – it is assumed that the description of dynamic uncertainty propagation demands different metric attributes than otherwise. The objective is hence to give the estimator not only the theoretical guidance on choosing the most suitable answers to the questions that need to be answered before commencing in the computational path for uncertainty quantification, but also to provide easy to use techniques for calculating these (including the easy identification, gathering and preparation of required data). Questions that may need to be explored in this process include the discovery of uncertainty propagation patterns, discovery of threshold influencers and dynamics and determination of rules for (dynamic) uncertainty quantification metric representation, i.e. the efficiency of various representational forms for uncertainty based on the reflection that the lower the information density the more alternatives to 2D-graph based representation might be suitable for sense-making.

Revisiting Zeno´s paradox from the research perspective we might therefore suggest rephrasing this as:

"If we estimate a probability field at a single instance in time it appears to have definite boundaries like a cloud in the sky on a windless day. How then do we describe the cloud on a windy day?"

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Appendices

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Definitions and abbreviations

Term	Definition
Advanced aerospace propulsion system (AAPS)	From the researchers´ perspective this is any airframe propulsion system not yet in current series production and which represents a step-change improvement on existing airframe propulsion system.
Anderson Darling (AD)	A statistical test used to assess the degree to which a sample data set follows a specific probability density function. It is commonly used to determine which type of probability density function most closely matches the distribution of the sample data set, whereby the test is deemed most appropriate for small numbers of sample data points.
Augmented data patterns (ADP)	Metrics related to patterns of data presented in augmented / immersive reality spaces. Due to the (dynamic) presentation of data in 3-dimensional and/or immersive spaces new opportunities are presented for pattern matching and recognition.
Autocorrelation (AC)	The cross-correlation of a data distribution with itself over sliding time-windows as a tool for finding repeating patterns.
Cellular automaton rules (AR)	Dynamic data arrays whose iteration patterns depend on specific rules governing the propagation behaviour of data points based on data array attributes (especially the behaviour / values of data point neighbours).
Bayes Risk (BR)	The minimum area of error due to overlapping decision boundaries of multiple probability density functions.
Beta coefficient (BC)	Describes the number of standard deviations a dependent variable may change as the predictor variables change. Often also called standardized co-efficient.
Business Value (BV)	From the researchers´ perspective an umbrella term describing all perspectives related to financial performance, i.e. earned value management, break-even, or value for money.

Central Limit Theorem (CLT)	Proposes that the larger the number of independently drawn data samples available, the more likely these are to follow a normal probability density function.
Co-efficient of dispersion (R^2)	Represents the proportion of variation in the response data which accepts regression analysis techniques. This metric is often also called the co-efficient of variation, co-efficient of determination or index of dispersion and is closely related to the concept of entropy.
Colors (CO)	From the researchers´ perspective the use of colours to indicate data values, i.e. traffic lights (red, amber, green) to communicate the status of a system.
Complexity (CM)	Describes the extent that a system is liable to exhibit emergent behaviour which is not predictable based on the understanding of its components.
Compression (CR)	Describes the extent that information can be encoded using less data than the source message. One metric example related to compression is that of statistical redundancy.
Conditional tail expectation (CTE)	A risk measure associated with the value at risk. Also known as tail value at risk.
Confidence Level (CL)	Describes the reliability with which a certain value can be found within a data set.
Correlation co-efficient (CC)	A metric describing the strength and direction of the vector relationship between two variables. Common measures are the Pearson product-moment, the Spearman or Kendall tau rank correlations, and the Goodman and Kruskal gamma values.
Cost Estimating Relationship (CER)	Describes the parametric interdependencies of variables affecting a cost estimate.
Cost Readiness Level (CRL)	A measure of the usability and quality of a cost estimate.

Cumulative distribution function (CDF)	Refers to the use of cumulated s-curves.
Data harmonics (DH)	Refers to the harmonics of data which has been sonified.
Deep uncertainty (DU)	A decision-making situation where Knightian uncertainty, conflicting divergent paradigms and emergent decision making are relevant, i.e. “The presence of one or more of the following three elements: (1) Knightian uncertainty: multiple possible future worlds without known relative probabilities; (2) Multiple divergent but equally-valid world-views, including values used to define criteria of success; and (3) Decisions which adapt over time and cannot be considered independently.” [85].
Defensible	From the researchers´ perspective the condition when an uncertainty estimate can be decomposed into a set of coherent elements which are realistic and understandable for experienced business decision makers.
Degrees of freedom (DF)	The minimum number of values which need to be specified to determine all the data points in a distribution.
Entropy (EP)	The dispersion of information across a probability field.
Estimated prediction error (EPE)	The three point uncertainty range associated with an unverified actual prediction error.
Fuzzy sets (FS)	Describes the relationships between data sets based on their degree of membership.
Half-life (HL)	Describes the time required for the accuracy of a metric to drop by 50%.
Homogeneity (HG)	Describes the degree to which assumptions regarding statistical properties can be applied across the probability field.
Interquartile range (IQR)	The range of values in a percentile, i.e. quartile.

Kurtosis (K)	A measure of the peakedness of a distribution.
Mean / median / mode (MEM)	The average and the middle values in a set of data.
Mean square error (MSE)	Describes the variance in a set of data after normalization based on differences in the means.
Minimax (MM)	The minimum and the maximum values / boundaries of a data range, whereby the “most likely” value is often included as a third reference point.
Minimum unbiased percentage error (MUPE)	An error regression metric helping to understand the relationship between individual observation error and magnitude of the observation.
Neural networks (NN)	A network structure of interdependent variables and commonly described by the composite metric of nonlinear weighted sum.
P-Value (PV)	The degree of statistical significance for an observed relationship.
Pedigree (P)	From the researchers´ perspective the measure of the degree of novelty.
Point	An estimate with zero uncertainty, i.e. at 100% confidence.
Probability (PR)	Probability and the related concept of likelihood describe the degree to which an event can be expected to take place.
Probability density function (PDF)	A function describing the distribution of continuous data in a probability field.
Probability Field (PF)	The range of values under consideration of deep uncertainty principles. The range can be described by a variety of metrics. Also referred to as uncertainty spaces, Hilbert spaces or hyper-spheres in the paper.
Quantification	The use of a numerical or visual metric to communicate the relative amount and pattern of data in a data set.
Range	The (dynamic) difference between an upper and a lower bound.

Rank Correlation (RC)	A measurement describing the degree of similarity between different rankings.
Risk	From the researchers´ perspective the probability of a predicted threat or opportunity occurring.
Root mean square deviation (RMSD)	Also referred to as the standard error of the mean and describes the relationship between the sample and population mean as the basis for creating confidence intervals.
RV Co-efficient (RVC)	Describes the closeness of two sets of points represented in matrix form.
Sample size (N)	The number of data points being analysed.
Shape (SH)	Variables characterizing the form of a function.
Sensitivity (S)	The degree of influence between inter-dependent factors.
Single point estimate (SPE)	A calculation with an uncertainty of "0", although it is common to add a small contingency of up to 5%.
Skew (SK)	Describes the difference between the left and right hand tails of a single modal distribution.
Smell (SM)	From the researchers´ perspective the use of olfactory approaches to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems remains hesitant.
Standard deviation (SD)	Describes the variance of a response based on statistical noise and is also called the standard error.
State-of-art	From the researchers´ perspective capabilities available for use in industrial practice.
State-of-past	From the researchers´ perspective capabilities historically used in industrial practice.
State-of-present	From the researchers´ perspective capabilities currently used in practice.
State-of-future	From the researchers´ perspective capabilities that are maturing towards use in industrial practice.

System of Systems (SoS)	A collection of interdependent (sub-) components which enables results no sub-part of the system can achieve on its own.
Statistics (STAT)	General statistical descriptions of data such as t-stat, f-stat, z-stat, or chi square.
Tactile quality (TQ)	From the researchers´ perspective the use of haptic approaches to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems is only progressing slowly outside of steering systems such as in aircraft.
Taste (T)	From the researchers´ perspective the use of gustatory senses to indicate data values. While this human sense plays a fundamental role in navigating and sense-making its transfer into purposeful communication and alert systems remains hesitant.
Technical Baseline (Cost) Estimate (TBE)	The single point engineering cost estimate that is input into the cost risk assessment process.
Technology Readiness Level (TRL)	A measure used to assess the maturity of a technology and scaled from basic technology research through to in-service operations.
Time criticality (TC)	The time for which an estimate is expected to maintain a certain accuracy or confidence.
Three point estimate (TPE)	An estimate which contains a worst, best and most likely value or boundaries.
Thresholds (TR)	Defines a step-change of a metric usually based on the switch of attractors.
Uncertainty	From the researchers´ perspective the single point actual prediction errors of the cost estimate.
Uniform density (UD)	The maximum entropy probability distribution for x in a normal distribution.

Uncertainty propagation (UP)	From the researchers´ perspective the actual iterative change in uncertainty of the TBE from the time of estimation to the time of verification.
Uncertainty quantification (UQ)	From the researchers´ perspective the process of determining the single point actual prediction error of a technical baseline estimates.
Volatility (V)	From the researchers´ perspective a measure used to describe the extent that data is expected to change over time intervals.

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